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БИФУРКАЦИЯ ЭНЕРГИИ СВЯЗИ И ХАОС В АТОМНЫХ ЯДРАХ

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В работе рассмотрена модель хаотического поведения нуклонов в атомных ядрах, построенная на основе модели ядерных взаимодействий и статистики Ферми-Дирака.

Ключевые слова: НЕЙТРОН, ПРОТОН, ФЕРМИ-ДИРАКА СТАТИСТИКА, ХАОС, ЭНЕРГИЯ СВЯЗИ, ЯДРО.

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BINDING ENERGY BIFURCATION AND CHAOS IN ATOMIC NUCLEI

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The model of chaotic behavior of nucleons in nuclei, based on the model of nuclear interactions and the Fermi-Dirac statistics is discussed.

Keywords: BINDING ENERGY, CHAOS, FERMI-DIRAC STATISTICS, PROTON, NEUTRON, NUCLEI.

It is known that the binding energy of nucleons in atomic nuclei depends on a regular motion of protons and neutrons in the nuclear shells, and on the chaotic behaviour of nucleons, which correlates with uncertainty in the measurement of the mass of the nuclides [1-3]. The concept of quantum chaos [4-5] is the basic model of chaotic behaviour of the nucleons.

We consider the model of the bifurcation of the binding energy in atomic nuclei, based on the generalized dynamics of the Verhulst-Ricker-Planck equation [6]. To derive the equations of the model the results of the theory of strong interactions of nucleons in nuclei [7-8] used. According to this theory there is a relationship between the size of the nucleus, binding energy and the interaction parameter, which can be written as follows:

$$r_n E = \sqrt{Sb_{nl}^{A}} = \beta(A)A \tag{1}$$

Here, $A = N + Z$ - the number of nucleons (protons + neutrons), as the units used the speed of light, Planck constant and electron mass. The binding energy is determined by the number of nucleons with a total mass of proton and electron, thus $E = A(m_p / m_e + 1) - M_A / m_e$.

Since equation (1) must be shared with the standard expression of the size of the nucleus, $r(A) = r_0 A^{1/3}$, reflecting the weak compressibility of nuclear matter, we can define the left-hand side of equation (1) using experimental data [9]. As a result, we find the product of the binding energy and nuclei radius depending on the number of nucleons - Fig. 1. For consistency with the data [9], we put

$\beta(A) = 0.05325 \ln A$.

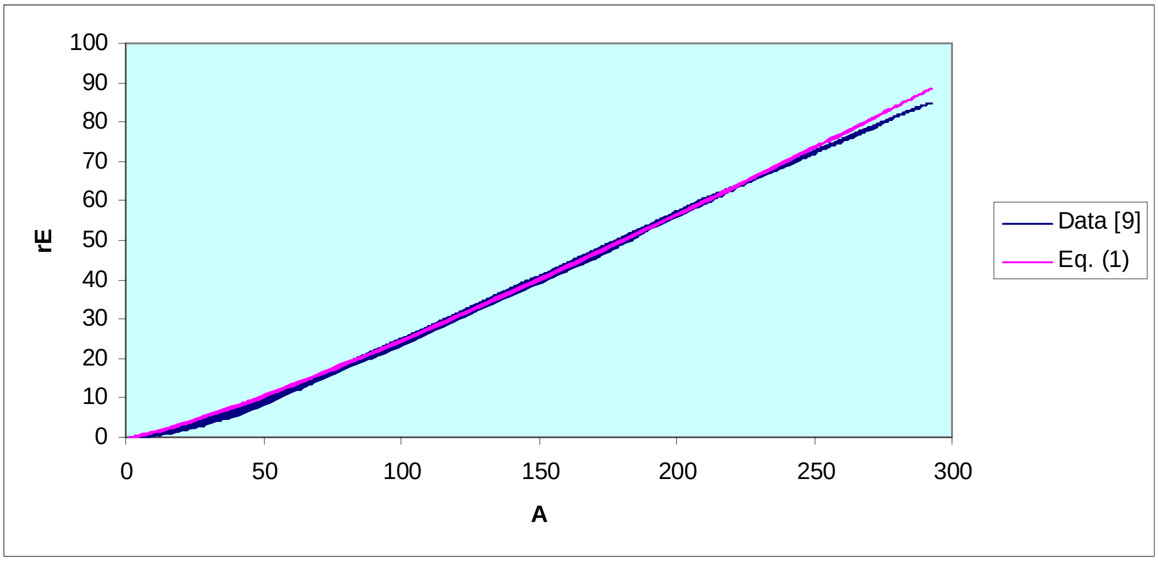

Fig. 1: The product of the binding energy and nuclei radius depending on the number of nucleons according to [9].

Using this correlation, we can represent equation (1) as

$$E_A = \beta A / r_A \tag{2}$$

Now we can construct a discrete model of the energy levels in nuclei as follows:

$$E_{A+1} E_A^2 = \frac{\beta(A+1)(A+1)(\beta(A)A)^2}{r_{A+1} r_A^2} = \frac{A}{4\pi r_A^3/3} \frac{4\pi r_A}{3 r_{A+1}} \beta(A+1)(A+1)(\beta(A)A)^2 \tag{3}$$

On the other hand, the density of nucleons can be related to the binding energy due to Fermi-Dirac statistics, we have

$$n_A = \frac{A}{4\pi r_A^3/3} = \frac{g_Z Z/A}{e^{(E_Z-\mu_Z)/\theta}+1} + \frac{g_N N/A}{e^{(E_N-\mu_N)/\theta}+1} \tag{4}$$

Here g_i, E_i, μ_i, θ are the weight factors, energy and chemical potential of protons and neutrons, and the statistical temperature of the nucleon, respectively. Model (3) - (4) was investigated in a wide range of parameters. Let us consider the results obtained in the simplified model under the condition of equality of chemical potentials of the two types of nucleons

$\mu_N = \mu_Z = \mu_A = \theta \ln \alpha$.

In this case, the model can be written as

$$\begin{aligned} x_{A+1}x_A^2 &= \frac{K(1+1/A)^{2/3}\beta^2(A)\beta(A+1)}{e^{-x_A}+\alpha} \\ x_A &= -\frac{E_A}{A\theta}, \quad K = \frac{4\pi}{3A\theta^3}\alpha g_A \\ g_A &= g_N + g_Z; \quad \beta(A) = 0.05325 \ \ln A \end{aligned} \tag{5}$$

To close the model (5), it is necessary to formulate the law of temperature and the weight factor change with the number of nucleons. We use a simple hypothesis, which follows from the theory of the Fermi gas of elementary particles [10] that these parameters are proportional to the cube of the boundary momentum, which in turn is determined by the size of the system:

$$\theta = k_1 p_0^3, \ g_A = k_2 p_0^3, \ p_0^2 = k_3 / r_A \tag{6}$$

Hence, we find that the temperature decreases with increasing number of nucleons as follows

$$\theta = \theta_0 A^{-1/2} \tag{7}$$

Under conditions (6)-(7), the parameter K on the right side of equation (5) does not depend on the number of nucleons. Let us consider the behaviour of the chemical potential depending on the number of nucleons. Above we assume that the chemical potentials of protons and nucleons are equal and, moreover, their relation to temperature is a constant, which coincides with the logarithm of the fine structure constant. To test this hypothesis, consider the function

$$f(A) = \mu_A / \theta \ln \alpha \tag{8}$$

Using data [9] and equations (5) - (7), we can define a function (8) - Fig. 2. Data [9] plotted in Fig. 2 show that the chemical potential of nucleons reaches the theoretical value $\mu_A = \theta \ln \alpha$ for the number of nucleons over 25. Therefore we can calculate a constant in eq. (7) as $\theta_0 = 50.7821 \quad \text{MeV}$.

Note two important facts that distinguish the bound nucleon system like as nuclei, from a gas of free fermions:

1) Chemical potential of the bound nucleon system is negative, whereas the chemical potential of free fermions is positive [10-11];

2) Chemical potential of the bound nucleon system varies linearly with increasing temperature, whereas the chemical potential of a system of free fermions decreases with increasing temperature and is limited by the Fermi energy at zero temperature - see [10-11].

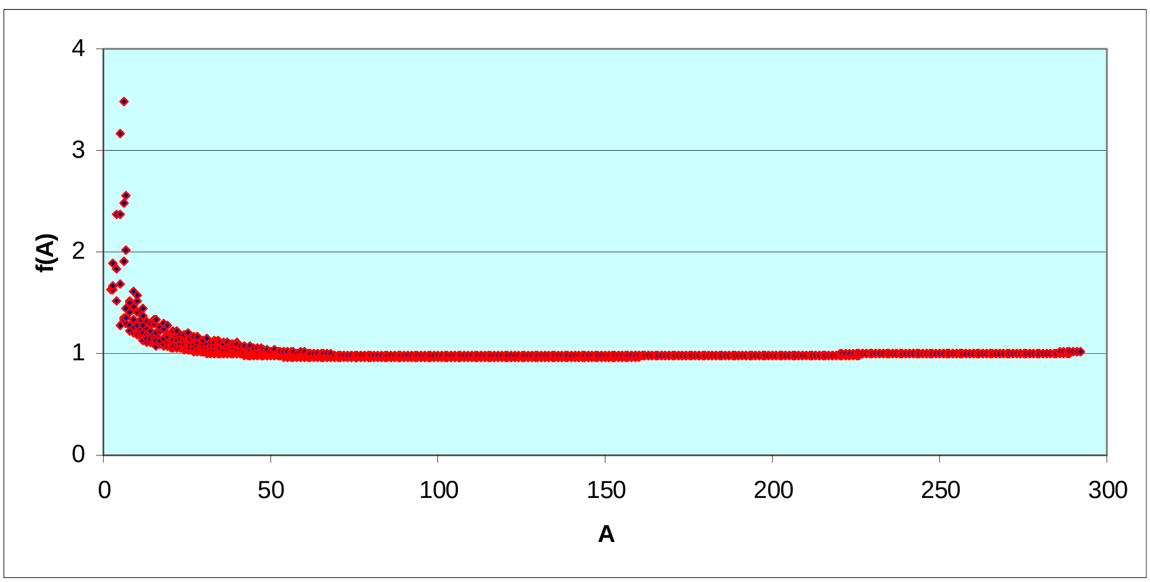

Fig. 2: The dependence of the chemical potential on the number of nucleons, calculated on equations (5) - (7), and according to [9]. $f(A) = \mu_A / \theta \ln \alpha$.

For light nuclei, the chemical potential, as well as other parameters of the model (5)-(7) deviates from the theoretical dependence (6). Nevertheless, we use the model (5), starting with the deuterium nucleus contains two nucleons.

We set the starting point at $x_2 = 0.2$. As a result, we find that the structure of energy levels, which is implemented in a system of nucleons - Figure 3. In this case, the first bifurcation point for the binding energy of light nuclei corresponds to the carbon isotope ^{12}C, and the second bifurcation point - nickel isotope ^{58}Ni. Note that in nature the regime with a lower binding energy is realized, although thermodynamic model (5) gives possible the second mode with a higher binding energy.

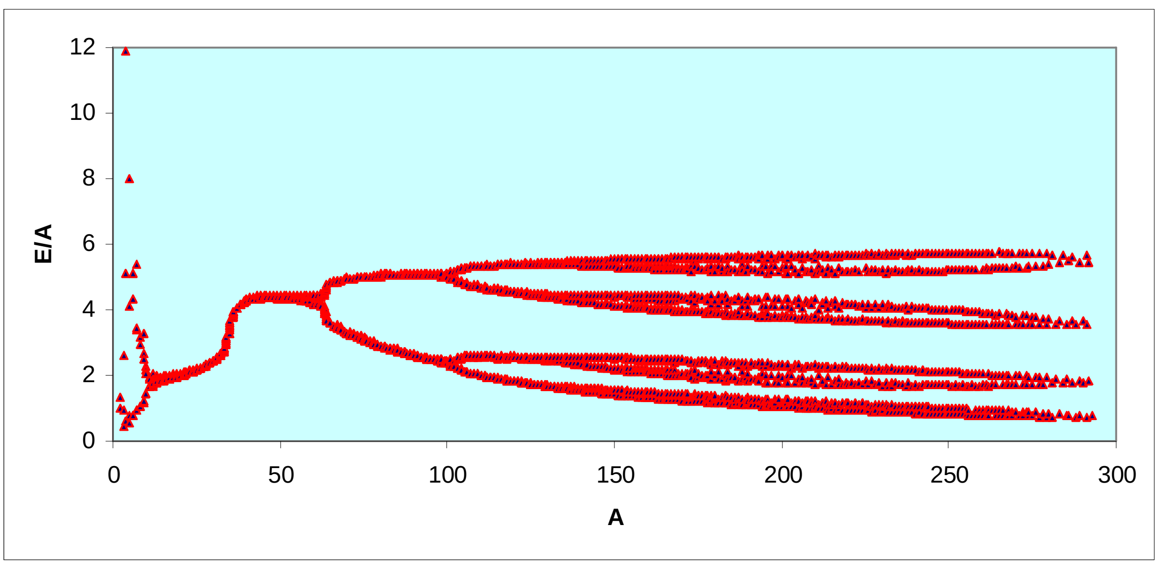

Figure 3: Bifurcation of the binding energy in the model (5), depending on the number of nucleons at $\alpha = 1/137$; $K = 0.0371$.

With the number of nucleon increasing the energy levels are split series at 2, 4, 8, 16 sublevels, as shown in Figure 3. The specific structure “four rats”, first observed in [6], is formed by increasing the parameter K - Figure 4. It was also shown in [6] that there is the transition to chaotic behaviour in a model (5) at

$\alpha = e^2 / \hbar c \leq 1/137$. It looks like the fine structure constant could be calculated from model (5) as a transition point between deterministic and chaotic behaviour of nucleons in a nuclei.

It was established that the transition to chaotic behaviour in a model (5) is also observed in violation of the equality of chemical potentials of the two kinds of nucleons – Figure 5. If the chemical potentials of protons and neutrons are strong differ, than the structure shown in Figure 6 forming, which superficially similar to the experimental dependence - Fig. 7.

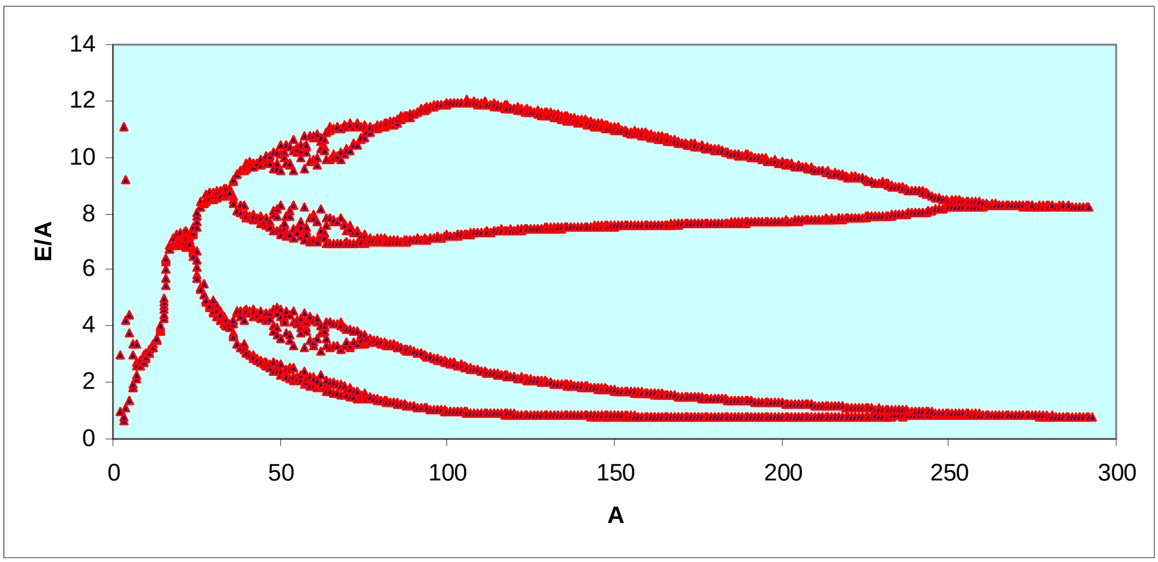

Figure 4: Bifurcation of the binding energy per nucleon in the model (5) depending on the number of nucleons at $\alpha = 1/137$; $K = 0.0839$.

Let us give an interpretation of the results. Model (3) - (7) is a thermodynamic one. It shows how the binding energy changing if one nucleon in the nuclei added, taking into account changes in density according to the Fermi-Dirac distribution at finite temperature and at large binding energy. It is well known that the binding energy of nucleons in the nucleus depends on the number of neutrons and protons. Standard semi-empirical formula describing the binding energy is given by [11]

$$E_b = a_1 A - a_2 A^{2/3} - a_3 Z(Z-1) A^{-1/3} - a_4 (N-Z)^2 A^{-1} + a_5 A^{-3/4} \quad (9)$$

$$a_1 = 14;\ a_2 = 13;\ a_3 = 0.585;\ a_4 = 19.3;\ a_5 = 33\,\delta(A, N, Z)$$

Here are shown current values of the coefficients derived from data [9]. All coefficients are given in MeV. In this expression, the function $\delta(A,N,Z)$ is defined as:

$\delta = 1$ for even Z, N;

$\delta = -1$ for odd Z, N;

$\delta = 0$ in all other cases.

The first and fourth term on the right side of expression (9) depend on the kinetic energy of nucleons, which is calculated on the basis of statistics (4) at zero temperature [11]. However, the data in Fig. 2 and eq. (6) - (8) show that temperature can have a finite value and the chemical potential can be varied other way than the theory of the Fermi gas of free particles predicts, like it explained in [10-11] and other university books. In particular, the chemical potential in a system of nucleons in nuclei is negative, as well as the binding energy.

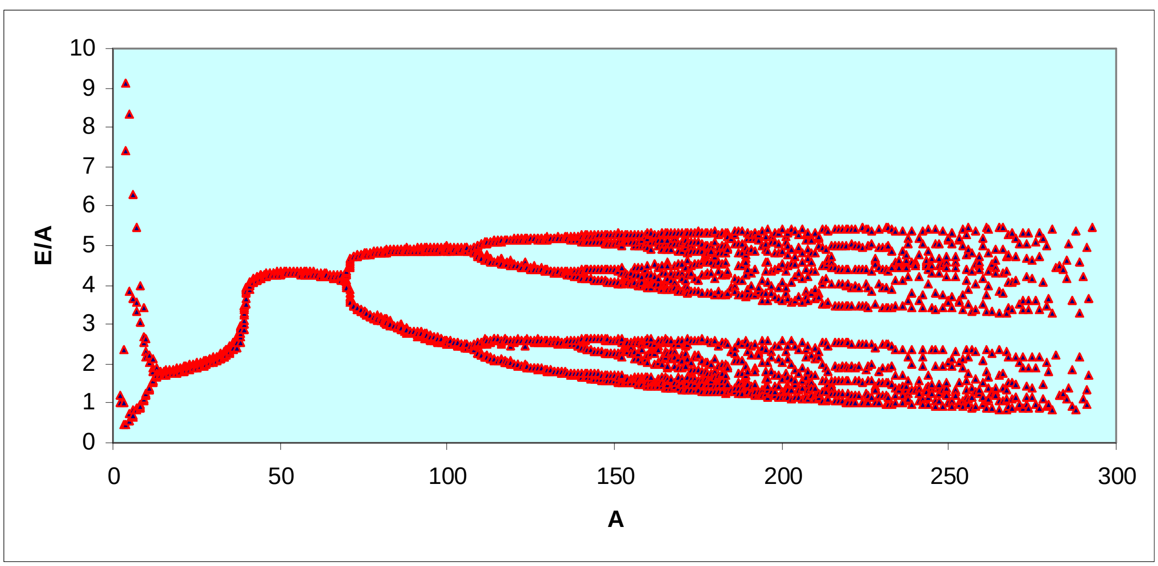

Figure 5: Bifurcation of the binding energy per nucleon and transition to chaotic behaviour at $K = 0.063;\ \mu_p / \theta = 1/137;\ \mu_n / \theta = 1/171$.

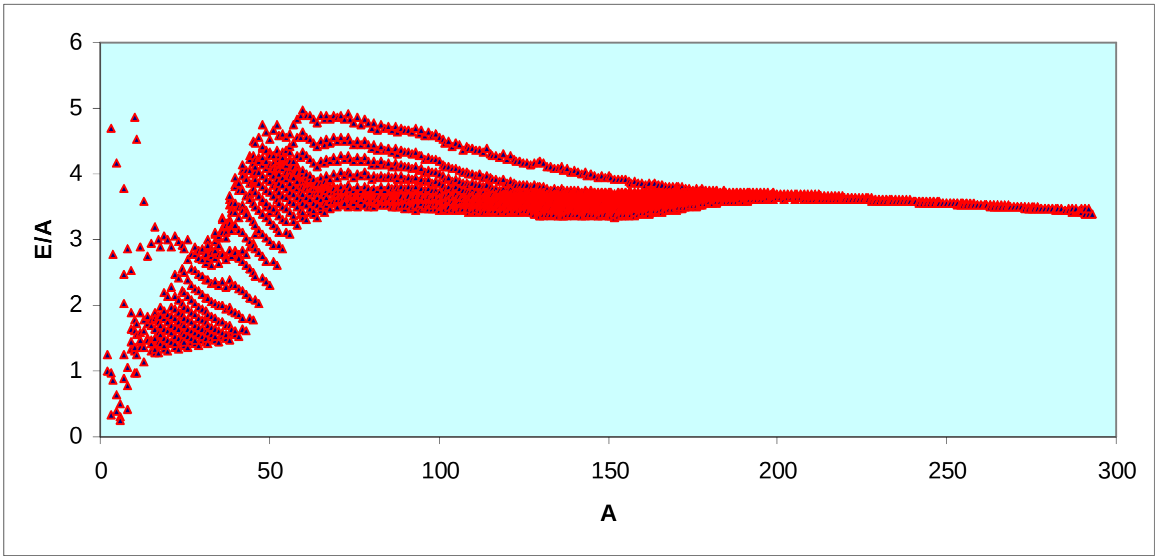

Figure 6: Bifurcation of the binding energy per nucleon and transition to chaotic behaviour at $K = 0.07$; $\mu_p / \theta = \alpha$; $\mu_n / \theta = \alpha^2$.

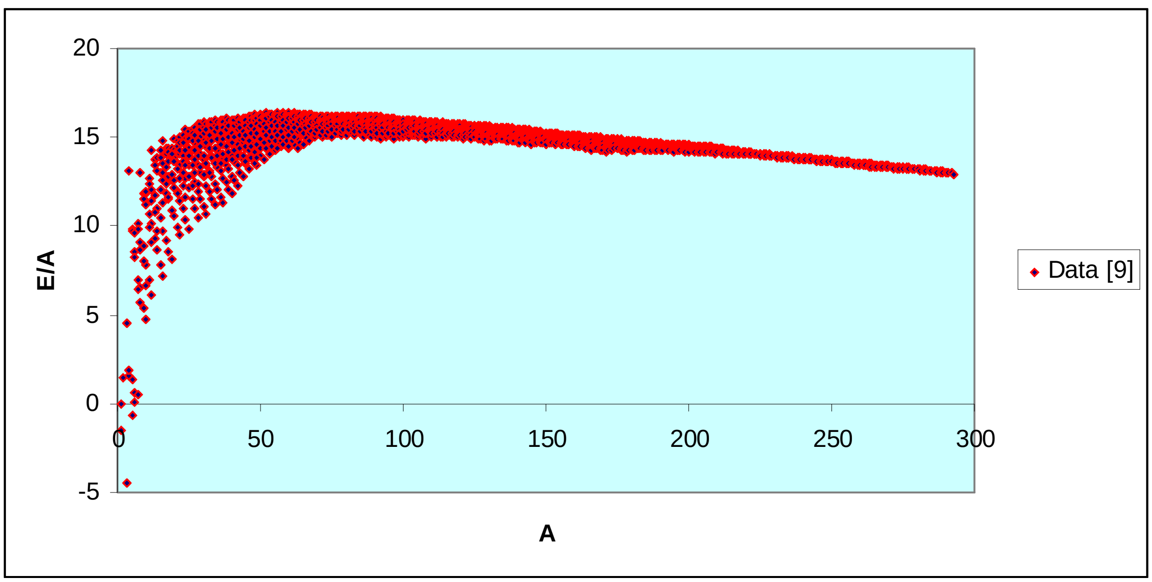

Figure 7: Binding energy per nucleon as a function of mass number according to [9].

Equation (5) is the simplest model describing the dynamics associated with changes in the number of fermions particles. Further studies will show whether it is possible to predict binding energy with accuracy exceeding the semi-empirical equation (9) based on model (3)-(4). The obtained results on the chaotic behaviour of the nucleons in the bound system indicate the complexity of describing the state of the nuclei, since the splitting of energy levels can occur not only due to the dynamic conditions imposed by the presence of nuclear interaction and the orbital motion, but also because of statistical reasons related to the influence of temperature in accordance with statistics of fermions.

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